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TABLES OF BOND VALUES—THEORY AND USE

By Montgomery Rollins,
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This article assumes that the material presented is to be referred to by the average practical dealer or investor in bonds, who seeks results for easy use and application. There are more exhaustive treatments of the subject[1] which may better serve the purpose of those engaged in the valuation of an estate, or in other instances where great care should be exercised in order that all parties may be treated equably.

It is strange how frequently one who has, during his entire business career, been familiar with the handling of investment securities, or, in fact, been in almost daily contact with such matters, fails to comprehend the principles upon which bond values tables are computed. The writer has been time and again surprised to find that men who should understand such matters suppose that it is a mere calculation by simple arithmetic, and that not to obtain the results given in the ordinary tables of bond values by their method astonishes them. Such people have begun on the supposition, to illustrate, that they could take a bond bearing 6 per cent interest, maturing in ten years, costing 110, and divide the premium—10 per cent—by the length of time which the bond has to run—in the case cited, ten years—and, obtaining 1 as the result, deduct it from 6 per cent, the rate which the bond bears, and assume, therefore, that the net return upon that particular investment is 5 per cent, the 10 per cent premium being charged off at the rate of 1 per cent yearly.

The failure in this reasoning arises from their not understanding the fundamental principles upon which such tables are based, which presuppose that the holder of a bond will, at the maturity of each one of the coupons, reinvest a sufficient portion of the money received, and keep it so invested until the maturity of the bond, so that the face value of the bond, added to the accumulation of reinvested interest, will, at its maturity, be exactly equivalent to the original cost of the same.

[1]See Chapter VIII of "The Accountancy of Investment," by Charles E. Sprague.

We have now arrived at the parting of the ways in this computation. There are two classes of accountants, or what you may choose to call them, whose ideas at this point sharply diverge. The first proceeds on the principle that the portion of the coupon money set aside shall be compounded at the same rate as the net return upon the investment. To illustrate: If it is a twenty-year 5 per cent bond, and selling at such a price as to yield 6 per cent, it is assumed that the money set aside shall be compounded at 6 per cent, regardless of the rates of interest which will probably prevail at such investing periods. To show further the absurdity of this, imagine the owner of several different lots of bonds, one lot having been purchased at a price to yield him 6 per cent per annum, another at 5 per cent, and another at 4 per cent. The class which we are now discussing assumes that a portion of these interest payments, even though they may all fall due at the same dates, shall be reinvested at compound interest at 6 per cent, 5 per cent, and 4 per cent respectively. It is unreasonable to believe that these three separate rates of interest will be ruling, at the same time for a similar grade of securities, or that there is any likelihood that the investor will guide himself, in the reinvestment of this interest, by taking into consideration the net return which he is enjoying upon the bonds in question.

The other school, which is undoubtedly the correct one, proceeds upon the plan of the reinvestment at some fixed definite per cent, say 3½ per cent or 4 per cent, without any regard to the net return which the original purchase price of the investment warrants. It does not take a very deep knowledge of finance to see that it is fairer to predict the future investment rates of money at some average rate, such as just mentioned, than at such widely divergent rates as by the other plan.

In the case of bonds selling at par both schools would be right as to their results, because there are no premiums or discounts to be provided for. Also, in the case of a bond computed by the first method, selling at a net return which is the arbitrary rate assumed as the reinvestment rate of the second method, then, likewise, will the two schools agree, but in all other instances they disagree. An idea of the amount of this disagreement may be shown by referring to the table accompanying this article, by which it will be seen that a 6 per cent bond having twenty years to run,

selling to net 5 per cent, is 112.55, and, in this case, 5 per cent is the compounding rate. By the use of a table of bond values based on a 4 per cent compounding rate, 112.03 is the result—a difference of nearly one-half of 1 per cent. Yet, custom has decreed, and undoubtedly always will, that the tables based upon the principles of the first school, including those of the author of this article, which he conceives to be inaccurate, are likely always to prevail in use, and that the tables of the second school will never reach any wide circulation. It would be relatively as great an undertaking in financial matters to change from the incorrect to the correct school, as it would be to introduce the metric system into this country, or to change the present standard gauge of railways.

In the circulars of banking houses offering investment securities, the financial columns of newspapers, and the "shop" talk of the investment dealer, will be encountered, with great frequency, such expressions as: "net return upon the investment," or, to be more specific, "a bond pays the investor 4½ per cent," "yields 4½ per cent," " is on a 4½ per cent basis," or whatever the rate may be. In any event, the intent is to convey the information as to what rate of interest the purchaser of a certain security at a given price may expect upon his money. By this is meant the proportionate rate which the income upon any investment bears to the total cost of that investment—"accrued interest" excepted—taking into consideration the time which it may be outstanding before being paid off.

Stocks, as a rule, are irredeemable, and consequently are figured as perpetual. Most bonds and other investments of a redeemable nature—having some fixed determinable time to run—are not so figured. A simple illustration of an irredeemable stock would be that of one selling at $200 per share, upon which dividends are being paid at the rate of 8 per cent yearly. In this case, the ratio of dividend, namely, 8 per cent, to the total cost, $200, would be 4, or 4 per cent, which is the investment yield. If the price of the stock were but $100 per share, and the dividend rate 4 per cent per annum, the yield would still be 4 per cent.

We now come to a security having some determinable date of maturity, and the problem likewise becomes more complicated. Special tables, commonly known as bond values tables, are used to ascertain the net returns from investments of this class. The books

comprising these tables are so arranged as to cover different periods for which redeemable securities are likely, in the experience of bankers, to be outstanding; and, therefore, cover half yearly periods from six months to, say, fifty years, and then at greater intervals to one hundred years, it being supposed that most securities of this class will mature in, perhaps, twenty-five or thirty years and the vast majority inside of fifty years.

To simply this article, page 45, which covers the twenty-year period, is reproduced from one of the ordinary books in use.

Henceforth, we shall speak of all redeemable securities as bonds. Let us now take an example of a bond having twenty years to run, bearing 5 per cent interest; at what price must it be sold to pay the investor 4 per cent? The twenty-year page above covers the period in question. The column headed 5 per cent must be taken and followed down until opposite 4 per cent in the extreme left-hand column. A result of 113.68 will be found, which is the rate of purchase of a bond to yield 4 per cent upon the investment; that is to say, $1,136.80, plus the interest which may have accrued since the last maturing coupon. This 4 per cent net return means 4 per cent per annum for each of the twenty years, and is reckoned upon the entire sum invested "accrued interest" excepted—or in this case, $1,136.80.

The time upon which to compute the net return, or the price of the bond, is the time from the date of computation to the maturity of the issue, not from the date of the issue, as some inexperienced persons have occasionally supposed, unless, of course, the date of issue and the date of computation should coincide.

This seems a pertinent place to consider at some length the matter of "accrued interest" referred to above. Strange as it may seem, there are many investors who fail to comprehend a subject, which, to most, is so simple. It is customary to make nearly all bonds with interest payable twice yearly. Let us take a $1,000 bond bearing 5 per cent interest. Upon this there will be found two coupons of $25 each, and, we will say, for the sake of simplicity, that these coupons fall due, one in January and the other in July of each year. On the first day of September, a purchase is made of a twenty-year bond at 113.68 and accrued interest. The purchaser will pay $1,136.80, which is the principal and premium, but in addition thereto, he will pay the interest upon $1,000, the face

20 YEARS

Interest Payable Semi-Annually.

PER CENT PER AN.	3%	3½%	4%	4½%	5%	6%	7%
2.90	101.51	109.06	116.60	124.15	131.70	146.80	161.89
3.	100.00	107.48	114.96	122.44	129.92	144.87	159.83
3.10	98.52	105.93	113.34	120.75	128.16	142.98	157.81
3⅛	98.15	105.55	112.94	120.33	127.73	142.52	157.31
3.20	97.06	104.41	111.75	119.09	126.44	141.13	155.82
3¼	96.34	103.66	110.97	118.28	125.59	140.21	154.83
3.30	95.63	102.91	110.19	117.47	124.75	139.30	153.86
3.35	94.93	102.17	109.42	116.66	123.91	138.40	152.89
3⅜	94.58	101.81	109.04	116.27	123.49	137.95	152.41
3.40	94.23	101.44	108.66	115.87	123.08	137.51	151.93
3.45	93.54	100.72	107.90	115.08	122.26	136.62	150.98
3½	92.85	100.00	107.15	114.30	121.45	135.74	150.04
3.55	92.17	99.29	106.41	113.52	120.64	134.87	149.11
3.60	91.50	98.58	105.67	112.75	119.84	134.01	148.18
3⅝	91.16	98.23	105.30	112.37	119.44	133.58	147.72
3.65	90.83	97.88	104.94	111.99	119.04	133.15	147.26
3.70	90.17	97.19	104.21	111.24	118.26	132.30	146.35
3¾	89.51	96.50	103.50	110.49	117.48	131.46	145.44
3.80	88.86	95.82	102.78	109.74	116.70	130.63	144.55
3⅞	87.90	94.81	101.73	108.64	115.56	129.39	143.22
3.90	87.58	94.48	101.38	108.28	115.18	128.98	142.78
4.	86.32	93.16	100.00	106.84	113.68	127.36	141.03
4.10	85.09	91.86	98.64	105.42	112.20	125.76	139.32
4⅛	84.78	91.54	98.31	105.07	111.84	125.37	138.90
4.20	83.87	90.59	97.31	104.03	110.75	124.19	137.63
4¼	83.27	89.96	96.65	103.35	110.04	123.42	136.80
4.30	82.68	89.34	96.00	102.66	109.33	122.65	135.98
4⅜	81.80	88.42	95.04	101.65	108.27	121.51	134.75
4.40	81.51	88.11	94.72	101.32	107.93	121.14	134.35
4½	80.35	86.90	93.45	100.00	106.55	119.65	132.74
4.60	79.22	85.72	92.21	98.70	105.19	118.18	131.16
4⅝	78.94	85.42	91.90	98.38	104.86	117.82	130.77
4.70	78.11	84.55	90.99	97.43	103.86	116.74	129.61
4¾	77.57	83.98	90.39	96.80	103.20	116.02	128.84
4.80	77.02	83.40	89.79	96.17	102.55	115.32	128.08
4⅞	76.22	82.56	88.90	95.24	101.59	114.27	126.95
4.90	75.95	82.28	88.61	94.94	101.27	113.92	126.58
5.	74.90	81.17	87.45	93.72	100.00	112.55	125.10
5.10	73.86	80.09	86.31	92.53	98.76	111.20	123.65
5⅛	73.61	79.82	86.03	92.24	98.45	110.87	123.29
5.20	72.85	79.02	85.19	91.36	97.53	109.87	122.22
5¼	72.34	78.49	84.64	90.78	96.93	109.22	121.51
5.30	71.85	77.97	84.09	90.21	96.33	108.57	120.81
5⅜	71.11	77.19	83.27	89.36	95.44	107.60	119.77
5.40	70.87	76.94	83.01	89.07	95.14	107.28	119.42
5½	69.90	75.92	81.94	87.96	93.98	106.02	118.06
5⅝	68.72	74.68	80.64	86.59	92.55	104.47	116.38
5¾	67.57	73.46	79.36	85.26	91.15	102.95	114.74
5⅞	66.43	72.27	78.11	83.95	89.78	101.46	113.13
6.	65.33	71.11	76.89	82.66	88.44	100.00	111.56
6⅛	64.25	69.97	75.69	81.41	87.13	98.57	110.01
6¼	63.19	68.85	74.51	80.18	85.84	97.17	108.50
6⅜	62.15	67.76	73.36	78.97	84.58	95.79	107.01
6½	61.14	66.69	72.24	77.79	83.34	94.45	105.55
6⅝	60.14	65.64	71.14	76.64	82.13	93.13	104.12
6¾	59.17	64.62	70.06	75.50	80.95	91.83	102.72
6⅞	58.22	63.61	69.00	74.39	79.78	90.57	101.35
7	57.29	62.63	67.97	73.31	78.64	89.32	100.00

value of the bond, from July first, when the last coupon was detached, until September first, two months. The bond bearing 5 per cent, this interest will be computed at that rate, and the investor will pay, in addition to the $1,136.80, $8.33, which is the interest on $1,000 for two months at 5 per cent. An investor may fail to comprehend that this $8.33 is not thrown away. As a matter of fact, it is returned to him when the next coupon is paid, which will be, following this illustration, January first. The investor will have held the bond four months, at the end of which time he will receive not only 5 per cent per annum for the time he will have held it, but, likewise, the $8.33 which he paid to the holder from whom he made the purchase. He will be out, however, interest on the $8.33 for the four months.[2] Here is where bonds and stocks sell differently, although there are exceptions to this rule. When a stock is sold, a sufficient price is added to the quotation so that it offsets the amount of interest—dividend—which has accrued since the last payment. A stock selling ordinarily at $100 a share and paying dividends at the rate of 4 per cent per annum, 2 per cent, say, each January and July, would, everything else being equal, be quoted at 101 half way between the two dividend periods, as the 1 per cent premium would fairly represent the dividend accumulation for three months at the rate of 4 per cent per annum.

On the New York Stock Exchange bonds are sold in this same way, and quotations include the interest accrued. Upon the Boston Stock Exchange they are—income or defaulted bonds excepted—sold plus the accrued interest, and the difference is here accounted for in the quotations upon the two different markets of the same security. The ordinary bankers selling bonds not listed upon the New York Stock Exchange, customarily sell the same "with accrued interest."

The foregoing explains such common expressions as "103 and accrued interest," "109 and accrued interest," or "109 and interest."

[2]The loss of interest upon the interest brings up the point that ordinary investment transactions always ignore this loss. Unless a bond by chance happens to be purchased upon a coupon date there must be some accrued interest paid, and absolute accuracy in figuring would demand the taking of this into consideration and would change slightly the net yield if it were figured into the actual purchase price, even though it were returned to the purchaser at the next coupon period. This would complicate matters so much, however, that it is seldom taken into consideration, as the amount, which is always against the purchaser and in favor of the seller, is slight.

An expression something like this is often encountered: "Yielding 4 per cent for the first ten years and 5 per cent for all time thereafter which the bond may run." By this it is understood that the issuer of the bond has the right to redeem it any time after ten years, but shall not be obliged so to do until some later date, as, in this case, twenty years. These bonds are known by such titles as "10-20 year bonds" or "10-20's," by which it is understood that they are absolutely due and payable in twenty years, but optional on the part of the issuer to redeem any time—generally upon a coupon date—between ten and twenty years. In a case of this kind, the seller must not assume that the bond will run longer than ten years. The greater the length of time which any form of an indebtedness, selling at a premium and having a fixed rate of interest, may be outstanding, the greater the percentage in interest return to the holder, prices always being equal. Therefore, in selling a 10-20 year bond at a premium, the net return should be computed on the basis of its being outstanding the minimum possible number of years—in this case ten—but should it run twelve years, for instance, before being paid off, the purchaser would benefit by the two additional years. That is, if the net return were computed, as it should be, on the ten-year basis, for any additional time which the bond might run, the investor would obtain a yield of the full rate of interest borne by the bond.

Should a bond be selling at a discount, the shorter the length of time which it runs the greater the interest return, prices being equal; the contrary to a bond selling at a premium. In computing the interest return or yield, the following rule must be observed, if the issue is "optional," so called, as in the case just cited:

Rule for Computing Net Yield of Optional Bonds

When bonds are selling at a premium, the interest return must be computed upon the shortest possible time which the security may be outstanding; when selling at a discount, the greatest length of time which it may be outstanding must be used as the basis.

In buying an issue of bonds known as "serials," that is to say, with a certain portion of the issue maturing periodically, many dealers in investment securities, who should know better, make the mistake of averaging the life of the issue, and then, by the use of a

table of bond values, basing their computation upon this average maturity; whereas, a separate price should be computed for each maturity, and then the average price taken—supposing, of course, that it is the intention to make one price for the entire lot, covering the different maturities. If bonds are bought by the first method and retailed by maturities, either a loss will result, or a less profit than expected.

The fallacy of averaging the maturity, and upon the period of time resulting computing the net return, arises from the fundamental principles set forth elsewhere in this article, that the net return upon a bond is based upon reinvesting at compound interest a certain portion of the coupons as they severally become due. Consequently, each maturity of a serial issue must be computed upon its own time, in order that this principle of compounding the interest may have true application.

An investor should guard against deceiving himself as to the income upon a redeemable bond, for which he has paid some price other than par. Let us illustrate by taking a bond having twenty years to run, bearing 5 per cent interest, and for which payment has been made at the rate of 113.68; that is, $1,136.80 and accrued interest; the net return by the ordinary bond values tables being 4 per cent. That is to say, the investor is supposed to receive 4 per cent per annum upon the purchase price of $1,136.80. In actual practice, as the coupons fall due, the investor receives $25.00 each six months, or $50.00 per annum. When the bond matures, he will receive, in addition to the last interest payment, only the principal sum of $1,000. There is, therefore, $136.80 premium paid that must be accounted for in some manner. A sinking fund, so called, may be set aside each half year out of the interest as received to provide for this premium. The investor is entitled to reckon his income at 4 per cent per annum on $1,136.80, the total purchase price, which would be $45.47, or, for each six months' period, one-half that sum; namely, $22.74. Deducting this from the semi-annual coupon leaves $2.26, which sum, if invested each six months at 4 per cent, will, at the maturity of the bond, added to the principal sum, equal $1,136.80, the original purchase price.

This is all based on the supposition that the $2.26 above mentioned will be invested promptly when received twice yearly at the rate of 4 per cent per annum; in other words, that it will be

compounded at 4 per cent per annum. It may be that this is an unfair rate, that a lower rate, 3½ per cent, or the prevailing savings bank rate, would be a better one to choose. If such were the case, a proportionately larger sum would have to be set aside each six months to create a sufficient sinking fund.

So far, we have had but one period, *i. e.,* twenty years, together with a fixed net return and price. The amount of the sinking fund must necessarily vary with the change of any one of the three factors: time, net return, or price. We will change but one of these, the time. Take nineteen years as the life of the bond when purchased. The tables show the price of a 5 per cent, nineteen-year bond to net 4 per cent to be 113.22, or $1,132.20 for a $1,000 bond. Proceeding again, as above, we find 4 per cent on this sum to be, for a half year, $22.64, or $2.36 less than $25.00, the amount of the six months' coupon. Here, then, is $2.36 for a nineteen-year bond, as against $2.26 for a twenty-year bond—prices and net return being equal—as the sinking fund.

The question naturally arises as to the way to treat similarly a bond bought at a discount. Let us again illustrate: A 5 per cent bond having twenty years to run, if purchased at the rate of 88.44, or $884.40 and accrued interest, will net the investor 6 per cent; that is, 6 per cent on the $884.40 invested. As the coupons fall due, he obtains, the same as in the above case, $25.00 each six months, or $50.00 per annum. When the bond matures he will receive, in addition to the interest, the full principal sum of the bond, $1,000, for which he has paid but $884.40. There is, therefore, a difference here of $115.60, by which amount the purchaser will be apparently enriched at the maturity of his bond. If, however, he wishes to avail himself of the full 6 per cent net return, which is he entitled to receive, he must anticipate this difference of $115.60, which may be done in this manner: He is entitled to reckon his income at 6 per cent on the $884.40, the original purchase price, which, for each six months, would call for $26.53. The coupon which he detaches from his bond provides for but $25.00 of this. There is, consequently, the sum of $1.53, which he should receive, from some source, to make his full 6 per cent interest. He may anticipate the $115.60, above referred to, by taking from some other fund this $1.53 each six months. This represents the amount which, if invested at 6 per cent, the same net return as provided for in the

investment, will, at the maturity of the bond, added to the $884.40, just equal $1,000. It will be noticed, however, that in this instance, it is supposed that the $1.53 will be compounded at 6 per cent, and here again the fallacy of the customary method of compounding the reinvestment portion is emphasized, for it is not likely, nor supposable, that these sums can be compounded at 6 per cent. But, in this case, as the bond is bought at a discount, the investor will not be likely to deceive himself; for accepting an arbitrary compounding rate of 6 per cent is necessarily taking a less amount (in this case $1.53) than he would if it were compounded at a lower rate. To prove this, let us suppose that 4 per cent is taken as this rate. The investor might then allow himself $1.88 each six months to add to his $25.00, to provide himself with a 6 per cent net rate.

To explain one more point in this connection, and following the illustration above where $1.53 is taken each six months, and which must be taken from some other fund, is there not a loss of interest each time upon that amount until the maturity of the bond? Or, in other words, what provides for the interest on these sums? That comes back at the maturity of the bond, for it will be noticed that if $1.53 be multiplied by 40, the number of coupons, the sum equals $61.20. But $115.60 will be received at the end of twenty years, and the difference between these last two sums is $54.40. That is to say, $54.40 represents the compound interest on the $1.53 periodically taken and expended as income.

The above argument is based upon the supposition that a bond will be held until maturity, or that, in case it should be disposed of earlier, the price realized shall be such as to give a yield equivalent to that at the time of purchase. In other words, if a bond having twenty years to run, bearing 5 per cent interest, is bought at 113.68, *i. e.*, a 4 per cent basis, and is, at the end of five years, sold, it is supposed that the price shall be computed on a basis of the fifteen years which the bond still has to run, which, to give a 4 per cent basis, would be 111.20. Instead, however, the holder of such a security may sell it at a higher price than the equivalent basis. What, then, shall be done with this surplus or profit? This question has been considered many times by the courts, which have decided that this excess premium belongs to the principal and should not be considered as income. This is from the standpoint of trustees. The ordinary investor, however, may treat it as he likes, except, that

in order to ascertain whether or not he has made a profit, he must find the price for the equivalent basis, and compare it with the price received.

Loring's "A Trustee's Handbook" deprecates the practice of buying bonds at a discount to offset those purchased at a premium, and his reasoning is that the difference in price is not simply a question of interest, but more often one of security.

Bond values tables cannot cover all rates of interest and all maturities. Neither can they give every conceivable net yield. To have in one volume sufficient matter to cover all the possible results, which investors or bankers may desire to obtain in the course of their investing or business careers, would require a volume beside which the family Bible of old would pale into insignificance. The most likely called for and commonly desired results only can be given in a volume of moderate dimensions. Likewise, financial conditions change. At times, rates of interest between 3¼ and 3¾ per cent are the prevailing levels of high-grade securities. It was not many years since, that few bond dealers would have had the temerity to predict a long-continued interval during which high-grade securities could be purchased to net the investor in the neighborhood of 5 per cent. Yet such is the condition of affairs at the time of writing this article. This is stated to illustrate the difficulties with which the authors of tables of bond values have to contend, in order to meet the popular demand. Tables issued a few years ago during the prevailing low rates of money are of little value to-day, when the rates have so largely increased.

The fact, however, that a book of bond values does not give every result sought for need not deter the user thereof from making some attempt to secure the result desired by a simple use of mathematics. We will confine ourselves to the twenty-year page already referred to as an illustration.

Suppose it were desired to know the price at which a 5½ per cent bond should be sold to net the investor 4 per cent. In the 6 per cent column, opposite 4 per cent, will be found 127.36. In the 5 per cent column, directly to the left, 113.68. Add these two results together and divide by 2, and you have the result for a 5½ per cent bond.

The highest rate bond which the sample page covers is 7 per cent. Prices to cover an 8 per cent bond may be found by obtaining

the difference between those of a 7 per cent and 6 per cent rate and adding the result to the former. Example:

Price of a 20-year 7 per cent bond to net 5½ per cent....	$118.06
Price of a 20-year 6 per cent bond to net 5½ per cent....	106.02
Subtract	$12.04
Add price of 7 per cent bond	118.06
Price of 20-year 8 per cent bond to net 5½ per cent....	$130.10

By an understanding of all this, it will be clear that the results for a bond bearing any rate of interest may be quickly computed.

Again, suppose it is a 5 per cent bond having twenty years to run and that it is desired to find the price at which it will net the holder 4.05 per cent. The nearest results in the table here given to this are 4 per cent and 4.10 per cent net returns. Find the column headed 5 per cent; obtain the results for 4 per cent and 4.10 per cent; add them together, and divide by 2, and the result, near enough for all practical purposes, will be obtained. There will, however, be a very slight inaccuracy. If a result for 4.03 per cent were desired, it would be necessary to find the prices opposite 4 per cent and 4.10 per cent; subtract the lesser from the greater, divide by 10, which is the difference between 4 per cent and 4.10 per cent in the left-hand column, and then you obtain what the ratio of change is in price for each one-hundredth of one per cent increase in the net return. Multiply your result, to follow this example, by 3, and deduct it from 113.68, the price to net 4 per cent, and you obtain the price of a 5 per cent bond to net the investor 4.03 per cent per annum. This, again, is a rough mathematical calculation and not absolutely accurate, but a little understanding of such matters will enable one to form approximate and useful conclusions.

It is sometimes desired to ascertain what a bond will yield at a given price when sold "flat." By this expression it is understood that the purchaser pays no accrued interest. A twenty-year bond, bearing 5 per cent interest, with coupons maturing semi-annually, February and August, is offered for sale, April 1st, at 115 "flat." What does it pay? We must first find out how much interest

has actually accrued upon the bond. In this case, it is two months. This, then, must be brought into dollars and cents. Two months' interest at 5 per cent on $1,000 (360 days to the year) is $8.33. The price of the bond is 115: that is, $1,150 for a $1,000 bond. Deducting the $8.33 just mentioned, you have as a result $1,141.67, which brings the bond down to 114.167, or, rounding out the second place to the right of the decimal, 114.17. To put it in another form, 114.17 and accrued interest is equivalent to 115 "flat." By referring to the table under the column headed, 5 per cent, we find that 114.17 lies between 115.18, which is a 3.90 per cent basis, and 113.68, which is a 4 per cent basis. We deduct the lesser of these two figures from the greater and obtain 1.50, which represents the ratio of increase for each variation of ten one-hundredths of one per cent in the net return. That is, a 3.90 per cent basis is to a 4 per cent basis as 115.18 is to 113.68. Divide 1.50 by .10, the difference between 3.90 per cent and 4 per cent, and you get what the ratio of decrease in price is for one one-hundredth of one per cent, which would be 15. Now we deduct 114.17, which is the price given, from 115.18, the nearest higher price in the tables, and obtain as a result 1.01. Divide this by 15, the ratio of change in price for each increase of one one-hundredth of one per cent in the net return, and we get .067; by which we understand that 114.27, the price given, is less than 115.18, the next higher price in the tables, as .067 is the increase in net return over 3.90 per cent. Add, therefore, these two together, 3.90 and .06+ and we obtain 3.96+, which equals the approximate net yield, according to this example, of a bond selling at 115 "flat," which is the equivalent of 114.17 and "accrued interest."

To sum up:

Price of bond—"flat"	$115.
Deduct 2 months' interest	.833
Price of bond—"with interest"................	$114.167 or $114.17
Price of bond to yield 3.90 per cent............	115.18
Price of bond to yield 4 per cent..............	113.68
Difference in price to equal .10 difference in yield	$1.50
1/100 per cent difference in yield, therefore, equals	15

Price of bond to yield 3.90 per cent equals....	$115.18
Deduct price of bond in example	114.17
Difference	$1.01
Divide by 15, the difference in price equivalent to difference in yield for each $^1/_{100}$ per cent and get	.067
Add to	3.90
	3.96 +, the result desired.

Be it understood, however, that this result is not absolutely accurate. There will be a variation of one or more one-hundredths, but it is a rough-and-ready way to obtain a very close approximation to what a bond will pay under conditions that are given. By the above method, it will be understood how to obtain the net return at a given price, when the price varies from what is actually given in the tables used.

It is seldom that a security is purchased upon a coupon date, and when such is not the case, tables which cover only half yearly periods are only approximate and must be adjusted to the actual time which the bond runs before maturity. For example, take a bond with 19 years 8½ months to maturity bearing 5 per cent interest, to net 4 per cent. The twenty-year table gives 113.68, the 19½ year table, 113.45. Subtract and get .23 which equals the difference in price between 19½ and 20 years for a 5 per cent bond netting 4 per cent. Nineteen years 8½ months lies between these two periods, and is 2½ months longer than 19½ years. There being twelve half months in a half year, divide 23, found above, by 12 and get .01916. As 2½ months are 5 half months, multiply .01916 by 5 and get .0958, which is added to the price of the 19½ year bond.

Thus 113.45 and .0958 give 113.5458, or, rounding out, 113.55, which is close enough for practical purposes.

We have been so far discussing bond values tables based upon redeemable securities with interest payable semi-annually. There are many issues of bonds in existence bearing annual interest, far more than the average bond dealer or investor realizes. There are, likewise, many other issues, such as our government securities, which have interest payable quarterly. It is not fair, therefore,

to use a table of bond values based on semi-annual interest payments to compute the net return upon issues of bonds with interest payable in annual or quarterly instalments. The semi-annual bond values tables, as already explained, are based upon the theory that a portion of the coupon money, as received, will be reinvested twice a year, and the interest compounded. In a bond with the interest payable but once a year, this money can only be reinvested and compounded once a year. Likewise, in a quarterly table, it will be four times a year. To show the difference, let us take (again see the table) a 4 per cent bond having twenty years to run. At 114.96 it pays 3 per cent as a semi-annual bond. As a bond with interest payable annually the price would be 114.88, and as a bond bearing quarterly interest payments, the price would be 115.00. There are to be had, therefore, separate sets of tables to meet these requirements.